

The Planck blackbody energy density as a function of wavelength, here denoted  $u(\lambda)$ , is a "density" as the energy is an integral of this density function over the considered wavelength range:

$$\int_{\lambda_1}^{\lambda_2} u(\lambda) d\lambda$$

This is similar to the physical mass of an object being the integral of the (varying) mass density over (x, y, z) space and similar to probability for a continuous probability distribution being the integral of the probability density function over the considered range of the independent variable. The equation for  $u(\lambda)$  is complicated but does not need to be considered in what follows.

The maximum of this energy density function is found by finding when  $u'(\lambda)=0$ . Unfortunately, there is no exact solution to this in terms of the usual functions but a very good approximate solution obtained numerically is  $\lambda_{\max} = 2898 [\mu\text{m K}]/T$  (where T is the temperature in Kelvin). This is known as Wein's (Displacement) Law for wavelength.

But what about if we approach the problem from a frequency, rather than wavelength, point of view. Let  $v(f)$  indicate the Planck blackbody energy density as a function of frequency  $f$ . Since  $f\lambda = c$  (c being the speed of light), doing the substitution in the integral of  $\lambda = c/f$  and remembering to replace  $d\lambda$  by  $(d\lambda/df) df$  as learned in doing substitution in integral calculus, we get:

$$\int_{f_1=c/\lambda_1}^{f_2=c/\lambda_2} u(c/f) \frac{d\lambda}{df} df$$

So the Planck blackbody energy density as a function of frequency is seen to be  $v(f)=u(c/f) (d\lambda/df)=u(c/f) (-c/f^2)$ . That will be maximum when the derivative of it is 0 or when (by the derivative product rule and chain rule):  $u'(c/f) (-c/f^2)(-c/f^2)+u(c/f) (2c/f^3)=(c/f^3) [cu'(c/f)/f+2u(c/f)]=0$  or simply when  $cu'(c/f)/f+2u(c/f)=0$ . (If this was simply  $cu'(c/f)/f=0$ , we would have a maximum at the frequency corresponding to the wavelength when the energy density as a function of wavelength was maximum, as  $u'(c/f)$  would be 0. But it is not due to the second term.) Again, there is no exact solution in terms of the usual function but a very good approximate solution obtained numerically is  $f_{\max}=5.879 \times 10^{10} [\text{Hz/K}] \bullet T$ . This is known as Wein's (Displacement) Law for frequency.

In summary, the densities as functions of wavelength and frequency have different equation and thus different maximums. Yet when the integrals over a range of frequencies or wavelengths are properly done (doing the correct substitutions), the energy will be the same.

Comment: The blackbody energy at exactly one wavelength or frequency is zero, just like the mass at exactly one point is zero or the probability of exactly one value of a continuous probability distribution is 0. So to say there is maximum energy at a specific wavelength or frequency is an error, as those are both 0. We do know that if we take intervals when the densities are large, the energy obtained will be relatively high.